The game of life in a glider's frame of reference

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Abstract

We introduce our work in progress on formally capturing motile systems inside dynamical systems. Ultimately the goal of this work is to express formally what it means that a dynamical system contains another system inside of it that may also move around. We focus on the example of the glider in the game of life, as previously studied by Beer, which is not only motile but also distributed in the sense that it extends over multiple cells of the cellular automaton. We propose definitions that apply to a much more general class of dynamical systems than the game of life. We do this by transforming the dynamics so that the glider (or other motile system) stays at the origin and the world moves around it.

Introduction

We often want to think of living organisms, or (less ambitiously) gliders in a cellular automaton, as dynamical systems coupled to their environments, which are also dynamical systems. This is not straightforward to do, however, because in the case of a glider (and presumably also in the case of an organism) there is no fixed set of variables or cells that correspond to its state. Instead the glider moves around the grid, so the set of cells it is composed of changes over time. In a series of papers, Beer explored the specific case of a glider in the game of life, showing how concepts from autopoietic theory can be applied to it (Beer, 2004, 2014a) and characterising its 'cognitive domain', or the set of perturbations that can change its state without destroying it (Beer, 2014b). Here we present work in progress on generalising these ideas, to produce definitions that could be applied to any glider-like system in a much broader class of dynamical systems than the game of life. The only thing we assume about the dynamical system is that it has a specified group of symmetries. But note that the glider-like system has to be given and is not computed as a result of our approach.

Naively trying to capture the glider

Let us consider the famous example of the glider in the game of life cellular automaton. We would like to have some way to look at the state of the cellular automaton and extract just the state of a glider.

Possibly the simplest idea is to use a surjective function $f\colon X \to S$ mapping the CA configuration or state space to the glider state space S. However, this does not work. The reason for this is that multiple states of a glider can occur in one and the same CA configuration. For example in an otherwise empty configuration any glider and the same glider 17 time steps later can also be present together in the same configuration (fig. 1). The presence of a glider in one state does not exclude the possibility of the presence of a glider in another state. So there is no unique glider state that the function f could return in this case.

Hence we need some way to define the 'state' of a glider that accounts for the glider's motion over time. This is not completely straightforward because the glider's motion depends on its state (the *structural coupling* discussed by Beer, 2014b).

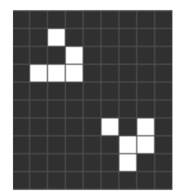

Figure 1: Two gliders in an otherwise empty CA state. The top left one will evolve into the bottom right one after 17 time steps.

There seem to be multiple related approaches to solve these problems. For this abstract we only discuss one. Instead of considering the global CA state and trying to pick out a glider, we transform the cellular automaton into a new dynamical system (no longer a CA) in which the glider remains at the centre and the world moves around it.

Apart from the problems of capturing a glider mentioned up to now there are also some additional challenges. One difficulty is that gliders can cease to exist, and any formalism capturing gliders should be able to deal with this. Another issue is that it is possible for one glider to split up into two gliders (for an example see Beer, 2014a, fig. 10). Ideally this would also be captured by the formalism. It turns out that the approach mentioned above can be extended to meet these challenges.

State-dependent frame of reference

Recall from Beer (2014b) that there are 16 possible configurations of the glider that can occur (centered) at any cell $v \in V$ of the CA. However, there are only two forms of the glider, the rocket R and the wedge W. If we start from the pair $\{R, W\}$ of a rocket and a wedge in one fixed direction and chirality, e.g. as they appear in fig. 1, we can obtain the other seven pairs by symmetry transformations. In what follows this allows us to define the *set of glider states* as $S \coloneqq \{R, W\}$, only containing the two forms.

Next, note that whenever there is a glider in one of the two forms $\{R, W\}$ centered at the cell $v \in V$ there cannot also be another glider in the other form at that cell. So for this cell we define a function $f\colon X \to S \cup \{\emptyset\}$ that maps the CA state to the according glider state or, if neither of the two forms of the glider in the according direction and chirality are present, to the empty set. We will derive functions that identify glider states in the other directions and chiralities from this function using symmetry operations.

Let G be the group of symmetries of the game of life cellular automaton and write $\sigma\colon G \times X \to X$ for its action. Then $\sigma(g_1 \cdot g_2, x) = \sigma(g_1(\sigma(g_2, x))$ by the definition of an action. We write, for example, $g_{\circlearrowleft}, g_{\rightarrow}, g_{\leftarrow}, g_{\uparrow}, g_{\downarrow}, g_{\updownarrow}$ for the group elements whose action rotates the state by $\pi/2$ counter-clockwise, shifts the state up, down, left, right by one cell, or mirrors it horizontally respectively.

Then, for example, the function that identifies gliders with opposite chirality that are going towards the top left to $S \cup \{\emptyset\}$ can be defined as $f \circ \sigma(g_{\circlearrowleft} \cdot g_{\updownarrow})$.

We can exploit the symmetry group further to maintain the *frame of reference of a given glider* on the CA state. For this note that given the entire CA state it is possible to determine the next CA state including whether there will be a glider and if so at which of the neighboring cells, in which direction, chirality and form it will occur. So assume there is a glider at v such that $f(x) = W$ and that in accordance with the rest of the CA state there will be a glider at the next time step (i.e. in CA state $h(x)$ where $h\colon X \to X$ is the update function of the entire CA) at cell $u \in V$ with direction $d \in \{\searrow, \nearrow, \nwarrow, \swarrow\}$ and chirality $\chi \in \{r, l\}$.

We can transform the state of the CA using a symmetry transformation such that this next glider is now centered at v again and has the same direction and chirality (but possibly a different state) as the original glider. In other words, we know there exists a group element $g \in G$ such that, if we update the state to $h(x)$ and then transform this next CA state by applying the associated action $\sigma(g, h(x))$ then the new glider will be a glider at v with the original glider's direction and chirality.

We can then define a function $\gamma\colon X \to G \cup \{\emptyset\}$ that always computes the next glider's position, direction and chirality from the current state of the system and returns the group element that transforms any next glider into the original glider's frame of reference (position, direction and chirality). If there is no next glider (for example within the lightcone of the current one as proposed by Beer, 2014a) then this function also returns the empty set.

Then define a new (partial) dynamical system on X as:

$$x_{t+1} = \sigma(\gamma(x_t), h(x_t)) \tag{1}$$

if $\gamma(x_t) \in G$. If $\gamma(x_t) = \emptyset$ then x_{t+1} is undefined. As long as a glider survives this dynamical system transforms the CA state around the glider while the glider only switches between rocket and wedge configuration. The glider state at x_t is $f(x_t)$. In fact, we can replace f and γ with a single function $\lambda\colon X \to (S \times G) \cup \{\emptyset\}$ giving us glider state and group element or (if the glider is dead) the empty set.

Using λ makes it easier to extend the formalism to deal with a glider splitting into two (or more). For this redefine $\lambda\colon X \to \mathcal{P}(S \times G)$ where $\mathcal{P}(Y)$ is the power set of a set Y. Then if the CA state indicates a glider will split into two λ can return two pairs $\{(s_1, g_1), (s_2, g_2)\} \in \mathcal{P}(S \times G)$, each containing a current glider state and a symmetry group element to transform the state to this glider's frame of reference. It could also return the empty set if there will be no glider since that is also an element of the power set. The partial dynamical system above then becomes a system whose states are sets of states of the CA and whose update function $h^{\mathcal{P}}\colon \mathcal{P}X \to \mathcal{P}X$ can be defined by

$$\begin{aligned} F_{t+1} =& h^{\mathcal{P}}(F_t) \\ =& \bigcup_{x \in F} \{\sigma(g, h(x)) \mid (s, g) \in \lambda(x)\} \end{aligned} \tag{2}$$

which generates for each state $x \in F_t$ the set or pairs (s, g) of glider points of view and for each one updates the state x and transforms it to that glider's frame of reference using the group action. Then it forms the union over the sets of next states generated from each state $x \in F$.

Ultimately our goal is to produce a formal framework in which gliders and their environments can be considered as coupled dynamical systems. One possibility is that this will allow concepts such as the viability boundary and perhaps even autopoiesis to be defined formally, in a way that generalises beyond the specific examples that have been studied up to now. Additionally, networks of coupled systems have received much recent attention in mathematics, particularly in the development of *categorical systems theory* (Myers, 2022); see (Niu and Spivak, 2023) for a related approach. Connecting our work to these frameworks would allow them to be used in reasoning about motile and distributed systems. Potentially, it could also allow our recent work on *interpreting* dynamical systems as doing Bayesian inference (Virgo et al., 2021; Virgo, 2023) or as solving POMDPs (Biehl and Virgo, 2023), to be applied to motile systems such as gliders.

In order to do this, we will need to consider what the glider system's inputs and outputs should be, generalising Beer's (2014b) formalisation of the cognitive domain. This will be the topic of future work.

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