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On the Edge of Chaos and Possible Correlations Between Behavior and Cellular Regulative Properties

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Abstract—**This paper gives an overview of the fundamental issues related to the classification of Cellular Automata (CA) classes. In particular, the possible locations of various CA capable to achieve different degrees of complex behaviors are described. This work is mainly focused on the correlation between CA behavior and cellular regulative properties. A possible minimalistic experimental setup of a development model is described, together with some ideas that can be further investigated in future work.**

1. Introduction

For the past 50 years computers have been based on the so called von Neumann architecture, where one complex processor sequentially performs a single task at each time-step. Recently, new computational paradigms have been explored and investigated. These new systems are based on a myriad of small and unreliable components called cells. Even if a single cell itself can do very little, the emergent behavior of the system as a whole is capable of complex dynamics. In cellular computing [12] each cell can only communicate with a few other cells, most or all of which are physically close by (neighbors). One implication of this principle is that none of the cells has a global view of the entire system, i.e., there is no central controller.

Such systems can be modeled using specific computational machines called cellular automata (CA). CAs are idealized versions of parallel and decentralized computing systems. Formally, a cellular automaton consists of a countable array of discrete sites or cells and a discrete-time update rule operating in parallel on local neighborhoods of a given radius. The metaphor with biology can be exploited on CAs because the physical structure is similar to biological multi-cellular organisms. For this reason, CA can also be used to abstract and simulate a biological development process. In Figure 1 a representation of the development of 1D CA is presented. The boundary cells are dealt by having the whole lattice wrap around into a torus, thus boundary cells are connected to “adjacent" cells on the opposite boundary.

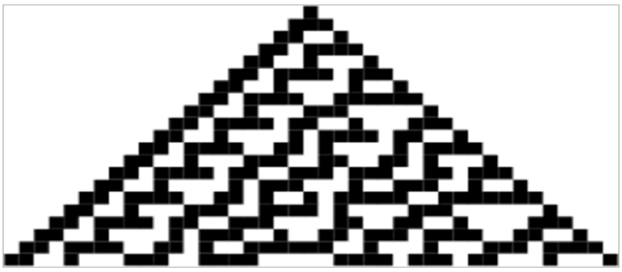

Fig. 1. 1D CA developed with rule 30 [15], represented in binary by 00011110

Wolfram [2] classified all CA behavior in four categories: class 1 evolves to a homogeneous state, class 2 to simple separated periodic structures, class 3 yields chaotic aperiodic patterns and class 4 yields complex patterns of localized structures. He also speculated that all class 4 CA rules have the capacity of universal computation. However class 4 is not rigorously defined and thus this hypothesis seems impossible to verify [13].

Later, Langton tried to find a relation between the CA behavior and a parameter λ [1]. He observed that the basic functions required for computation (transmission, storage and modification of information) are achieved in the vicinity of phase transitions between ordered and disordered dynamics (edge of chaos). He also guessed the location of Wolfram classes in the λ space. Classes 1 and 2 constitute the ordered phase and class 3 constitutes the disordered phase. Class 4 is located somewhere between these two phases of dynamical behavior.

Packard [3] and Crutchfield and Mitchell [4] [5], investigated the frequency of evolved CA rules as a function of Langton's λ parameter. However, the results differed.

In Section II, Wolfram's CA classes are described. In Section III, Langton's work on the computation at the Edge of Chaos is presented and in Section IV other related work is described. Section V discusses a possible minimalistic experimental setup for the investigation of correlation between CA behavior and cellular regulative properties. Finally, Section VI concludes the paper.

2. Wolfram's Qualitative CA Classes

John von Neumann [8] studied the first cellular automaton in the 1940s, basing his research also on some studies done by Stanislaw Ulam [9]. At that time, the research on cellular automata was performed through analysis of variations produced by a single automaton. For example, between 1960 and 1970, John Conway's Game of Life automaton [11] was studied extensively to understand the behavior of specific CA rules capable of such complex dynamics. The basic technique used for investigation was to find specific rules to define an automaton and then study the generated patterns. Moreover, all those studies were performed using two-dimensional cellular automata. In the 1980s Stephen Wolfram changed this approach. He showed that one-dimensional cellular automata could be sufficient to investigate the rules' behavior. Instead of studying single rules, he divided and enumerated subclasses of CA rules in order to group the rules producing similar behavior and study different classes depending on the pattern that the rules were able to generate [2]. Wolfram found out that CA can be grouped in four classes, depending on the type of behavior they produce:

- Class 1: Evolution leads to homogeneous state;
- Class 2: Evolution leads to a set of separated simple stable or periodic structures;
- Class 3: Evolution leads to a chaotic pattern;
- Class 4: Evolution leads to complex localized structures, sometimes long-lived.

All the different cellular automaton behaviors that can be developed and evolved can be associated with one of the previous classes.

In practice, with finite lattices, there is only a finite number of possible configurations and all rules lead to a periodic behavior. However, in theory, the lattice is supposed to be infinite. The number of possible CA configurations grows exponentially with the increase in the size of the automaton and the number of cell types. For example, whilst a 1D CA of size 16 and 2 cell types has 2^{16} (=65536) possible states, a 2 dimensional CA of size 16 by 16 and 3 cell types has 3^{256} (~$1{,}39 \times 10^{122}$) possible states.

2.1. Class 1

Almost all initial configurations relax after a transient period to the same fixed configuration. In this class, the outcome of the evolution is determined with probability 1 and it is not dependent on the initial state.

In other words, the automata in this class die after one or few evolution steps and the process is not reversible because all the previous information is lost. No patterns, or very few patterns, are produced in the first evolution steps and the CA reaches homogeneous state every time. This is represented in Figure 2, where the top line corresponds to the initial state and each subsequent row shows one development step.

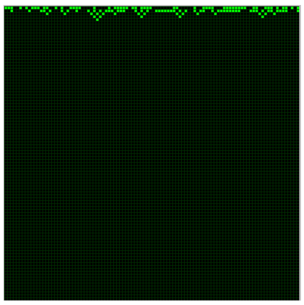

Fig. 2. Example of class 1 CA rule. (1-dimensional cellular automaton with random initial configuration)

2.2. Class 2

Almost all initial configurations relax after a transient period to some fixed point or some temporarily periodic cycle of configurations, which is dependent on the initial configuration.

Some parts of the initial configuration are filtered out and others are propagated forever. The automata in this class will look like vertical bars, as in Figure 3, or stair steps. The process is not reversible because information is partially lost during the evolution.

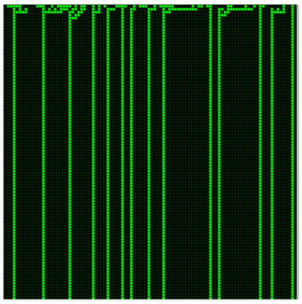

Fig. 3. Example of class 2 CA rule. (1-dimensional cellular automaton with random initial configuration)

2.3. Class 3

Almost all initial configurations relax after a transient period to chaotic behavior.

The evolution process is completely reversible since the previous state can be predicted analyzing the current state. This class of behaviors is chaotic but not random and the produced data are not noise. A CA is considered reversible if and only if, for every current configuration of the CA, there is exactly one past configuration. Figure 4 presents an example of class 3 behavior.

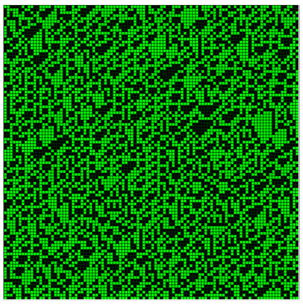

Fig. 4. Example of class 3 CA rule. (1-dimensional cellular automaton with random initial configuration)

2.4. Class 4

Some initial configurations result in complex localized structures that are sometimes long-lived.

The information is propagated by the automata at variable speed. The process is non-reversible because the current site values could have arisen from more than one previous configuration. This is the only class with non-trivial automata, which can produce complex behaviors instead of fixed dynamics (trivial by definition) or chaotic dynamics (chaotic behavior is considered to be trivial because it is not random and thereby it is completely reversible). The only way of finding the value of a specified cell after a certain number of time steps is to actually compute the whole evolution, otherwise the problem is intractable.

In Figure 5, a one dimensional CA initialized with a random configuration illustrates a class 4 behavior.

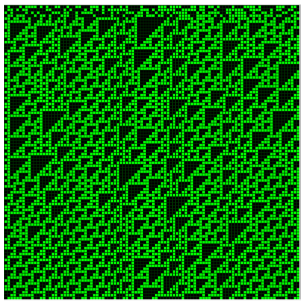

Fig. 5. Example of class 4 CA rule. (1-dimensional cellular automaton with random initial configuration)

Wolfram [2] proposed that the automata in the class 4 are capable of universal computation (any computable problem can be solved by it in a finite number of time steps). It is not possible to guess how many time steps a certain problem will require to be solved (this issue is similar to the Halting Problem [14]).

3. Computation at the Edge of Chaos

The Edge of Chaos refers to a region in the CA rule space where there is a phase transition between ordered and chaotic behavioral regimes.

Langton [1] hypothesized that it is more likely to find rules capable of complex computation in a region where the value of a parameter (λ) is critical. λ is defined as follows:

$$\lambda = 1 - \frac{q}{tot} \qquad (1)$$

In (1), the ratio *(q/tot)* represents the fraction of “non-quiescent” states in the rule-table used for the development of the CA. When λ is equal to zero all the rules lead to a quiescent state and when λ is equal to (2) the rule-table is the most heterogeneous (k represents the number of different possible states of a cell).

$$\lambda = 1 - \frac{1}{k} \qquad (2)$$

For example, considering a 2D CA with 3 cell types (empty cell, cell of type A and cell of type B), if all the rules constructed with all the possible neighborhood configurations lead to the empty cell, the value of λ is 0. On the other hand, if only 1/3 of the rules generate the empty cell, the value of lambda is 1-1/3=2/3.

Incrementing λ from 0 to 1-1/k it is possible to observe all the types of CA behavior described by Wolfram, crossing phase transitions and going from ordered behavior to chaotic behavior. For certain λ critical values the CA tend to show complex and long-lived patterns.

Langton [1] claimed also that Wolfram’s class 4, the one capable of universal computation, is located somewhere around this critical value of λ, at the Edge of Chaos. It may be said that in such a region the basic conditions to support computation (information transmission, storage and modification) are probably present [1]. This is represented in Figure 6.

It turned out also that, at intermediate values of λ in the proximity of a phase transition, the behavior of the CA is complex and unpredictable, while in other areas of the λ spectrum the behavior seems simple and easy to predict.

Moreover, Langton studied entropy as a measure of the information carried by each cell during the CA development and mutual information between the cell and itself at the next time step. His research supports the hypothesis that for λ values in the proximity of phase transitions, it is more likely to have well balanced conditions to support computation. For example, information storage involves low entropy and, on the other hand, information transmission requires increasing entropy. If a system needs to do both in order to perform computation, there must be a tradeoff between levels of entropy. Again this happens in the proximity of phase transitions.

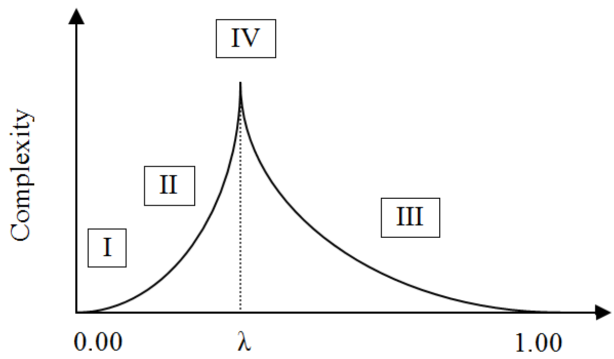

Fig. 6. Location of the Wolfram classes in λ space [1]. Class 4 is represented between classes 2 and 3.

4. Other Related Work

Packard [3] tried to evolve CA rules that were capable of performing a specific task using genetic algorithms [10]. From his experimental results, he suggested that the most computationally effective rules that are selected by the evolutionary process lie near the transition to chaos.

Mitchell et al. [4] tried to reproduce the same experiments performed by Packard obtaining different results. They suggested that the original results were due to mechanisms in the particular genetic algorithm used in the experiment rather than intrinsic computational properties of λ near the transition to chaotic regime.

5. Discussion and Possible Experimental Setup

Summarizing the paper, it can be said that more investigation is required in order to understand the relation between computation performed by CA and different λ values, the relationship between complexity and computation, the intrinsic properties of the development process of CAs. So far, it seems that there is a correlation between phase transitions in the CA regime and the ability to perform computation.

A developmental system is a system in which an organism can develop (e.g. grow) from a zygote to a multicellular organism (phenotype) according to specific local rules, represented by a genome (or genotype), and the interactions with the environment. A CA can be considered as a developing organism, where the developmental process is itself a computation. The genome specifications and the gene regulation information control the cells' growth and differentiation. The behavior of the CA is represented by the emergent phenotype, which is subject to size and shape modifications, according to the cellular changes along the developmental process. Such dynamic developmental systems can show adaptation, self-modification or plasticity properties.

Our current work consists on an investigation with a minimalistic developmental model based on a two dimensional cellular automata. The number of cell types is set to three instead of two in order to keep the property of multicellularity (two types of cells plus the empty or dead cell). A single cell is placed in the centre of the development grid and it will develop using a developmental table based on Von Neumann's neighborhood (four cells orthogonally surrounding the central cell). All the possible regulatory input combinations are explicitly represented in a development table which consists of 243 (3^5) configurations (with three types of cells and a neighborhood of size five, the number of possible cellular states is 243). With this configuration it is possible to investigate the developmental process from a zygote to a multicellular organism with development rules that cover a wide spectrum of the λ space. Depending on the λ value, it may be possible to find a correlation between properties of the developmental mapping and the behavior of the automata in terms of developmental complexity, structural complexity, CA attractor length, CA trajectory length and different transient phases.

In many studies regarding CA and in particular their development process and the produced computation, artificial organisms have shown remarkable abilities of self-repair, self-regulation [6] and phenotypic plasticity [7].

Our guess is that many of the CA rules which lead to these behaviors are in the frozen or ordered region (Wolfram's classes 1 or 2) and not in the "edge of chaos".

6. Conclusions

In this paper, Wolfram's CA behavior classification is described together with the "edge of chaos", a region in the rule space where there is a transition between ordered and chaotic regimes. An experimental setup for a minimalistic developmental system that can be exploited to investigate correlations between CA behaviors and cellular regulative properties is presented.

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